ISBN: 9781844805747

Chapter 18 *Mathematical background*

Proof of AP

Proof that

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

The sum of an arithmetic progression with n terms may be written

$$S_n = a + (a + d) + (a + 2d) + \cdots + [a + (n - 1)d]$$

This expression may also be written as

$$\begin{aligned} S_n = {} & [a + (n - 1)d] + [a + (n - 1)d - d] \\ & + [a + (n - 1)d - 2d] + \cdots + (a + d) + a \end{aligned}$$

by reversing the order of the terms on the right-hand side of the equation.

Adding the two equations gives

$$\begin{aligned} 2S_n &= [2a + (n - 1)d] + [a + d + a + (n - 1)d - d] \\ &\quad + \cdots + [a + (n - 1)d + a] \\ &= [2a + (n - 1)d] + [2a + (n - 1)d] + \cdots \end{aligned}$$

but as there are n terms

$$2S_n = n[2a + (n - 1)d]$$

and thus,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

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Proof of GP

Proof that

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

The sum of a geometric progression with n terms may be written:

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

multiplying both sides by r gives

$$rS_n = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n$$

Subtracting the first equation from the second gives

$$rS_n - S_n = ar^n - a$$

$$S_n(r - 1) = a(r^n - 1)$$

So,

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

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Proof of quadratic formula

Proof that

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

The form of a quadratic equation is

$$ax^2 + bx + c = 0$$

Dividing by a and rearranging gives

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Adding $\frac{b^2}{4a^2}$ to both sides gives

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

This can be written

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Taking the square root of each side gives

$$x + \frac{b}{2a} = \pm\frac{\sqrt{(b^2 - 4ac)}}{2a}$$

Hence

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$